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# PBIB-DESIGNS AND MATRIX FORM OF ASSOCIATION SCHEMES ARISING FROM MINIMUM EDGE COVERING SETS OF SOME CIRCULANT GRAPHS

**B. Chaluvaraju[1] and Shaikh Ameer Basha[2]**

[1,2]Department of Mathematics, Bangalore University, Jnana Bharathi Campus, Bangalore -560 056, INDIA

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## ABSTRACT

Let $G = (V, E)$ be a graph. The smallest number of edges in any edge cover of G and is called its *edge covering number* $\alpha_1$ In this paper, we obtain the total number of $\alpha_1$-sets, the Partially Balanced Incomplete Block (PBIB)-Designs on minimum $\alpha_1$-sets of Circulant graphs with *m*-association schemes for $1 \le m \le \left\lfloor \frac{p}{2} \right\rfloor$.

**KEYWORDS**: Edge cover, PBIB-Designs, matrix, association schemes & Circulant graphs.

## I. INTRODUCTION

All graphs considered in this paper are finite, undirected and connected with no loops and multiple edges. As usual $p = |V|$ and $q = |E|$ denote the number of vertices and edges at a graph $G$, respectively. For any undefined terms in this paper, we refer to [16].

For a given positive integer $p$, let $s_1, s_2, s_3, \ldots, s_t$ be a sequence of integers with $0 < s_1 < s_2 < \ldots < s_t < \frac{p+1}{2}$. The Circulant graph $C_p(S)$ where $S = (s_1, s_2, s_3, \ldots, s_t)$ is the graph on $p$ vertices labelled as $v_1, v_2, v_3, \ldots, v_p$ with vertex $v_i$ adjacent to each vertex $v_{i\pm si (mod\, p)}$ and the values $s_t$ are called jump sizes.

The Circulant graphs have been investigated in the fields outside of graph theory. For example, in geometers, Circulant graphs are known as star polygons [12]. They have been used to solve problems in group theory (particularly the families of Cayley graphs), as shown in [1] as well as number theory and analysis. The Circulant graphs and their matrices have important applications to the theory of designs and error correcting codes. They are also, used as models for interconnection networks in telecommunication, VLSI designs, parallel and distributed computing. For applications and mathematical properties of Circulant graphs, for more details, we refer to [3], [8], [17], [18], [19] and [20].

Bose and Nair [5] introduced a class of binary, equi-replicate and proper designs, which are called Partially Balanced Incomplete Block (PBIB)-Designs. In these designs, all the elementary contrasts are not estimated with the same variance. The variances depend on the type of association between the objects.

PBIB-Designs are widely used in Cryptology, digital fingerprint codes, many experimental situations such as the removal of lichens, weathering stability under ultraviolet radiations, an in-ground natural durablity field test and one can refer [11] and [12].

Given $v$ elements (objects or vertices), a relation satisfying the following conditions is called a PBIB with *m*-associate classes:

(i) Any two treatments are either first associates or second associates,. . . or $m^{th}$ associates, the relation of associations being symmetric.

(ii) Each object $x$ has exactly $n_k$, $k^{th}$ associates, the number $n_k$ independent of $x$.

(iii) If two objects $x$ and $y$ are $k^{th}$ associates, then the number of objects which are $i^{th}$ associates of $x$ and $j^{th}$ associates of $y$ is $p_{ij}^k$ and is independent of the $k^{th}$ associates $x$ and $y$. Also $p_{ij}^k = p_{ji}^k$.

With the association scheme on $v$ objects, a PBIB–Design is an arrangement of $v$ objects into $b$ sets (called blocks) of size $g$ where $g < v$ such that

(i) Every element is contained in exactly $r$ blocks.

(ii) Each block contains $g$ distinct elements.

(iii) Any two elements which are $m^{th}$ associates occur together in exactly $\lambda_m$ blocks.

The number $v$, $b$, $g$, $r$, $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_m$ are called the parameters of the first kind, whereas the numbers $n_1, n_2, \ldots, n_m, p_{ij}^k$ $(i, j, k = 1, 2, \ldots, m)$ are called the parameters of the second kind.

Bose [4] has initiated the study of strongly regular graph with parameters (*p, l, σ, μ*) of a finite simple graph on *p* vertices, regular of degree l (with $0 < l < p-1$, so that there are both edges and non edges), such that any two distinct vertices have $\sigma$ common neighbors when they are adjacent, and $\mu$ common neighbors when they are nonadjacent. For more details, we refer to [6], [7] and [15].

Covering number: For any graph $G = (V, E)$, a set of edges which covers all the vertices of $G$ is a edge cover for $G$, the smallest number of edge covering number $\alpha_1(G)$.

In this paper, $\tau(G)$is used to denote the maximum number of $\alpha_1$-sets of a graph $G$. For details on PBIB designs and its graph theoretic parameters, we refer to [9], [10], [11], [13], [14] and [21].

## II. CIRCULANT GRAPH $C_p(1)$

The graph $C_p(1)$ with jump size one with $p \geq 3$ vertices.

**Observation 2.1.** The Circulant graphs $C_4(1)$ and $C_5(1)$ are the strongly regular graphs.

**Proposition 2.1.** [16] For any Circulant graph $C_p(1)$ with $p \geq 3$ vertices,

$$\alpha_1\left(C_p(1)\right) = \left\lceil \frac{p}{2} \right\rceil$$

**Theorem 2.1.** The collection of all $\alpha_1$-sets of a Circulant graph, $C_p(1)$; $p = 2n+1$ , $n \geq 1$ vertices form PBIB-Designs with $\left\lfloor \frac{p}{2} \right\rfloor$-association scheme and parameters are $v = p$, $b = p$, $g = \left\lceil \frac{p}{2} \right\rceil$, $r = \left\lceil \frac{p}{2} \right\rceil$ and

$$\lambda_m = \begin{cases} \left\lceil \frac{m}{2} \right\rceil, & m \equiv 1 (mod\ 2) \\ \left\lceil \frac{p}{2} \right\rceil - \lambda_{m-1} & otherwise \end{cases}$$

**Proof.** Let $C_p$ (1) be a Circulant graph with $p \geq 4$ vertices and labelled as $v_1, v_2, v_3, \dots, v_p$ . By Proposition 2.1, we have $\alpha_1\left(C_p(1)\right) = \left\lceil \frac{p}{2} \right\rceil$. Further, $C_p$ (1) with $p \geq 3$ have $p$ blocks of $\alpha_1$-set, it implies $b = \tau\left(C_p(1)\right) = p$.

By Proposition 2.1, we obtain $g = \alpha_1\left(C_p(1)\right) = \left\lceil \frac{p}{2} \right\rceil$, where $g$ is the number of elements contained exactly in a block.

By virtue of the above facts, we get $r = \left\lceil \frac{p}{2} \right\rceil$. To obtain the $m$–associates for the elements, where $1 \leq m \leq \left\lfloor \frac{p}{2} \right\rfloor$. Two distinct elements are first associates, if they have jump size 1 and they are $k^{th}$ – associates $2 \leq k \leq \left\lfloor \frac{p}{2} \right\rfloor$, if they have $k$ jump sizes. These associates are as shown in Table 1 along with their matrix representations.

**Table 1.** Association schemes for Circulant graph with *even vertices*.

| Elements | Association scheme | | | | | |
|---|---|---|---|---|---|---|
| | First | Second | … | $k$ | … | $\frac{p}{2}$ |
| $v_1$ | $v_p, v_2$ | $v_{p-1}, v_3$ | … | $v_{p-(k-1)(mod\ p)}$, $v_{(k+1)(mod\ p)}$ | … | $v_{1+\frac{p}{2}}$ |
| $v_2$ | $v_1, v_3$ | $v_p, v_4$ | … | $v_{p-(k-2)(mod\ p)}$, $v_{(2+k)(mod\ p)}$ | … | $v_{2+\frac{p}{2}}$ |
| $v_3$ | $v_2, v_4$ | $v_1, v_5$ | … | $v_{p-(k-3)(mod\ p)}$, $v_{(3+k)(mod\ p)}$ | … | $v_{3+\frac{p}{2}}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $v_i$ | $v_{(i-1)(mod\ p)}$, $v_{(i+1)(mod\ p)}$ | $v_{(i-2)(mod\ p)}$, $v_{(i+2)(mod\ p)}$ | … | $v_{p-(k-i)(mod\ p)}$, $v_{(i+k)(mod\ p)}$ | … | $v_{(i+\frac{p}{2})(mod\ p)}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $v_p$ | $v_{p-1}, v_1$ | $v_{p-2}, v_2$ | | $v_{p-k}$ , $v_k$ | | $v_{\frac{p}{2}}$ |

By Table 1, the parameters of second kind are given by $n_i = 2$ for $1 \leq i \leq \frac{p-1}{2}$ and $n_{\frac{p}{2}} = 1$.

With the association scheme, we have the matrix representation of the Circulant graph; $p \geq 3$ is

$$P^k = \begin{pmatrix} P_{11}^k & P_{12}^k & \cdots & P_{1\frac{p}{2}}^k \\ P_{21}^k & P_{22}^k & \cdots & P_{2\frac{p}{2}}^k \\ \vdots & \cdots & \ddots & \vdots \\ P_{\frac{p}{2}1}^k & P_{\frac{p}{2}2}^k & \cdots & P_{\frac{pp}{22}}^k \end{pmatrix}$$

Therefore the possible values of $k$ in the matrix $P^k$ are given below :

If $k = 1$, then

(i) $p_{ij}^{l} = 1$ for $1 \le i \le \frac{p}{2} - 1$, $j = i + 1$,

(ii) $p_{ij}^{l} = 1$ for $i = 1 + j$, $1 \le j \le \frac{p}{2} - 1$.

If $2 \le k \le \frac{p}{2} - 1$, then

(i) $p_{ij}^{k} = 1$ for $1 \le j \le \frac{p}{2} - 1$, $i + j = k$, $j = k + i$ and $i + j = p - k$.

(ii) $p_{ij}^{k} = 1$ for $1 \le j \le \frac{p}{2} - 1$, $i = k + j$ and $i + j = p - k$.

If $k = \frac{p}{2}$, then $p_{ij}^{k} = 2$ for $1 \le i \le \frac{p}{2} - 1$ and $j = k - i$ with other entries are all zero.

Hence the parameters of first kind are given by $v = p, b = p, g = \left\lfloor\frac{p}{2}\right\rfloor, r = \left\lfloor\frac{p}{2}\right\rfloor$ and

$$\lambda_m = \begin{cases} \left\lceil\frac{m}{2}\right\rceil, & m \equiv 1(mod\ 2) \\ \left\lceil\frac{p}{2}\right\rceil - \lambda_{m-1} & otherwise \end{cases}$$

**Theorem 2.2.** The collection of all $\alpha_1$-sets of a Circulant graph $C_p(1)$; $p = 2n$, $n \ge 2$ vertices form PBIB-Designs with $\left\lfloor\frac{p}{2}\right\rfloor$- association scheme and parameters are $v = p$, $b = 2$, $g = \frac{p}{2}$, $r = 1$and $\lambda_m = \begin{cases} 1, & m \equiv 0(mod\ 2) \\ 0, & otherwise \end{cases}$

**Proof.** Let $C_p(1)$ be a Circulant graph with $p \ge 4$ vertices and labelled as $v_1, v_2, v_3, \ldots, v_p$. By Proposition 2.1, we have $\alpha_1\left(C_p(1)\right) = \left\lfloor\frac{p}{2}\right\rfloor$. Further, $C_p(1)$ with $p \ge 4$ have $p$ blocks of $\alpha_1$-set, it implies $b = \tau(C_p(1)) = 2$ . By Proposition 2.1, we get $g = \alpha_1\left(C_p(1)\right) = \frac{p}{2}$, where $g$ is the number of elements contained exactly in a block. By virtue of the above facts, we have $r = 1$. To obtain the $m$ – associates for the elements, where $1 \le m \le \left\lfloor\frac{p}{2}\right\rfloor$. Two distinct elements are first associates, if they have jump size 1 and they are $k^{th}$ – associates $2 \le k \le \left\lfloor\frac{p}{2}\right\rfloor$, if they have $k$ jump sizes. These associates are as shown in Table 2 along with their matrix representations.

Table 2. Association schemes for circulant graphs odd vertices.

| Elements | Association scheme | | | | | |
|---|---|---|---|---|---|---|
| | First | Second | … | $k$ | … | $\frac{p}{2} - 1$ |
| $v_1$ | $v_p, v_2$ | $v_{p-1}, v_3$ | … | $v_{p-(k-1)(mod\ p)}$, $v_{(k+1)(mod\ p)}$ | … | $v_{1+\frac{p-1}{2}}, v_{1+\frac{p-1}{2}+1}$ |
| $v_2$ | $v_1, v_3$ | $v_p, v_4$ | … | $v_{p-(k-2)(mod\ p)}$, $v_{(2+k)(mod\ p)}$ | … | $v_{2+\frac{p-1}{2}}, v_{2+\frac{p-1}{2}+1}$ |
| $v_3$ | $v_2, v_4$ | $v_1, v_5$ | … | $v_{p-(k-3)(mod\ p)}$, $v_{(3+k)(mod\ p)}$ | … | $v_{3+\frac{p-1}{2}}, v_{3+\frac{p-1}{2}+1}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $v_i$ | $v_{(i-1)(mod\ p)}$, $v_{(i+1)(mod\ p)}$ | $v_{(i-2)(mod\ p)}$, $v_{(i+2)(mod\ p)}$ | … | $v_{p-(k-i)(mod\ p)}$, $v_{(i+k)(mod\ p)}$ | … | $v_{(i+\frac{p-1}{2})(mod\ p)}$, $v_{(i+\frac{p-1}{2}+1)(mod\ p)}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $v_p$ | $v_{p-1}, v_1$ | $v_{p-2}, v_2$ | | $v_{p-k}, v_k$ | | $v_{\frac{p-1}{2}}, v_{\frac{p-1}{2}+1}$ |

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With this association scheme, the parameters of second kind are given by $n_i = 2$ for $1 \le j \le \frac{p-1}{2}$ and

$$P^k = \begin{pmatrix} P^k_{11} & P^k_{12} & \cdots & P^k_{1\frac{p-1}{2}} \\ P^k_{21} & P^k_{22} & \cdots & P^k_{2\frac{p-1}{2}} \\ \vdots & \cdots & \ddots & \vdots \\ P^k_{(\frac{p-1}{2})1} & P^k_{(\frac{p-1}{2})2} & \cdots & P^k_{(\frac{p-1}{2})(\frac{p-1}{2})} \end{pmatrix}$$

The possible values of $k$ in the matrix $P^k$ are given below:
If $k = 1$, then

(i) $p^1_{ij} = 1$ for $1 \le i \le \frac{p-1}{2} - 1$, $j = i + 1$,

(ii) $p^1_{ij} = 1$ for $1 \le j \le \frac{p-1}{2} - 1$, $i = 1 + j$,

(iii) $p^1_{ij} = 1$ for $i = \frac{p-1}{2}$, $j = \frac{p-1}{2}$.

If $2 \le k \le \frac{p-3}{2}$, then

(i) $p^k_{ij} = 1$ for $1 \le i \le \frac{p-3}{2}$, $i + j = k$, $j = k + i$ and $i + j = p - k$,

(ii) $p^k_{ij} = 1$ for $1 \le j \le \frac{p-3}{2}$, $i = k + j$ and $i + j = p - k$.

If $k = \frac{p-1}{2}$, then

(i) $p^k_{ij} = 1$ for $1 \le i \le \frac{p-3}{2}$, $j = \frac{p-1}{2} - i$,

(ii) $p^k_{ij} = 1$ for $1 \le i \le \frac{p-1}{2}$, $j = \frac{p-1}{2} - i$

With other entries are all zero.
Hence the parameters of first kind are given by $v = p$, $b = 2$, $g = \frac{p}{2}$, $r=1$ and

$$\lambda_m = \begin{cases} 1, & m \equiv 0 (mod\ 2) \\ 0, & otherwise. \end{cases}$$

## III. CIRCULANT GRAPH $C_p\left(\frac{p}{2}\right)$.

The Circulant graph with jump size $\left\lfloor\frac{p}{2}\right\rfloor$; $p \ge 3$ vertices, is $C_p\left(\frac{p}{2}\right)$.

**Proposition 3.1.** [16] For any Circulant graph $C_p\left(\left\lfloor\frac{p}{2}\right\rfloor\right)$ with $p = 2n + 1$, $n \ge 1$ vertices, $\alpha_1\left(C_p\left(\left\lfloor\frac{p}{2}\right\rfloor\right)\right) = \left\lceil\frac{p}{2}\right\rceil$.

**Theorem 3.1.** The collection of all $\alpha_1$-sets of a Circulant graph $C_p\left(\left\lfloor\frac{p}{2}\right\rfloor\right)$, $p = 2n + 1$, $n \ge 1$vertices form PBIB-Designs with $\left\lfloor\frac{p}{2}\right\rfloor$- association scheme and parameters are $v = p$, $b = p$, $g = \frac{p}{2}$, $r = \left\lceil\frac{p}{2}\right\rceil$ and

$$\lambda_m = \begin{cases} \left\lceil\frac{m}{2}\right\rceil, & m \equiv 1 (mod\ 2) \\ \left\lceil\frac{p}{2}\right\rceil - \lambda_{m-1} & otherwise \end{cases}$$

**Proof.** Let $C_p\left(\left\lfloor\frac{p}{2}\right\rfloor\right)$ be a Circulant graph with $p \ge 3$ vertices and labelled as $v_1, v_2, v_3, \dots, v_p$. By Proposition 3.1, we have $\alpha_1\left(C_p\left(\left\lfloor\frac{p}{2}\right\rfloor\right)\right) = \left\lceil\frac{p}{2}\right\rceil$.

Further, $C_p\left(\left\lfloor\frac{p}{2}\right\rfloor\right)$ with $p \ge 3$ have $p$ blocks of $\alpha_1$-set, it implies $b = \tau\left(C_p\left(\left\lfloor\frac{p}{2}\right\rfloor\right)\right) = p$.

By Proposition 3.1, we have $g = \alpha_1\left(C_p\left(\left\lfloor\frac{p}{2}\right\rfloor\right)\right) = \left\lceil\frac{p}{2}\right\rceil$., where $g$ is the number of elements contained exactly in a block.

From the above facts, we have $r = \left\lceil\frac{p}{2}\right\rceil$.

To obtain the $m$ – associates for the elements, where $1 \le m \le \left\lfloor\frac{p}{2}\right\rfloor$.

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Two distinct elements are first associates, if they have jump size $k$ and they are $\left\lfloor\frac{p}{2}\right\rfloor^{th}$ – associates $1 \leq k \leq \left\lfloor\frac{p-2}{2}\right\rfloor$, if they have $\left\lfloor\frac{p}{2}\right\rfloor$ jump sizes. These associates are as shown in Table 2 along with their matrix representations. Hence the parameters of first kind are $v = p$, $b = p$, $g = \frac{p}{2}$, $r = \left\lfloor\frac{p}{2}\right\rfloor$ and

$$\lambda_m = \begin{cases} \left\lfloor\frac{m}{2}\right\rfloor, & m \equiv 1 (mod\ 2) \\ \left\lfloor\frac{p}{2}\right\rfloor - \lambda_{m-1} & otherwise \end{cases}$$

## IV. CIRCULANT GRAPH WITH ODD JUMP SIZES

The Circulant graph with odd jump size $C_p\left(1,\ 3, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$; $p \geq 6$ vertices.

**Observation 4.1.** The sequence of an odd jump size from 1 to $\left\lfloor\frac{p}{2}\right\rfloor$, the $C_p\left(1,\ 3, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$ is strongly regular graph.

**Proposition 4.1.** [16] For any Circulant graph $C_p\left(1,\ 3, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$ with $p \geq 6$ vertices,
$\alpha_1\left(C_p\left(1,\ 3, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)\right) = \left\lfloor\frac{p}{2}\right\rfloor$.

**Remark 4.1.** $C_p\left(1,\ 3, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$ with degree $\delta \geq 4$ doesn't form PBIB-Designs. Since the parameters $b$ is not equi-replicate and $r$ is distinct.

## V. CIRCULANT GRAPH WITH EVEN JUMP SIZES

The even jump sizes $\left(2,\ 4, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$ of Circulant graph is denoted by $C_p\left(2,\ 4, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$with $p \geq 4$ vertices.

**Observation 5.1.** For $C_p\left(2,\ 4, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$; $p = 2n$, $n \geq 2$ vertices leads to disconnected components, so we are not considering these Circulant graphs.

**Observation 5.2.** The Circulant graphs $C_5(2)$, $C_6(2)$, $C_8(2,4)$, $C_{10}(2,4)$, $C_{12}(2,4,6)$ are few examples of strongly regular graphs.

**Proposition 5.1.** For any Circulant graph $C_p\left(2,\ 4, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$; $p \geq 4$ vertices,
$\alpha_1\left(C_p\left(2,\ 4, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)\right) = \left\lfloor\frac{p}{2}\right\rfloor$.

**Remark 5.1.** $C_p\left(2,\ 4, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$ with degree $\delta \geq 4$ doesn't form PBIB-Designs. Since the parameters $b$ is not equi-replicate and $r$ is distinct.

## VI. CIRCULANT GRAPH $C_p\left(1, 2, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$.

The jump size of Circulant graph is $1, 2, \dots, \left\lfloor\frac{p}{2}\right\rfloor$ with $p \geq 3$ vertices.

**Proposition 6.1.** For any Circulant graph $C_p\left(1, 2, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$ with $p \geq 3$ vertices,
$\alpha_1\left(C_p\left(1, 2, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)\right) = \left\lfloor\frac{p}{2}\right\rfloor$.

**Remark 6.1.** $C_p\left(1, 2, \dots, \left\lfloor\frac{p}{2}\right\rfloor\right)$ with degree $\delta \geq 4$ doesn't form PBIB-Designs. Since the parameters $b$ is not equi-replicate and $r$ is distinct.

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